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Monopole Loop Suppression and Loss of Confinement in Restricted Action SU(2) Lattice Gauge Theory

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Abstract

The effect of restricting the plaquette (1×1 Wilson loop) to be greater than a certain cutoff is studied. The action considered is the standard Wilson action with the addition of the plaquette restriction, which does not affect the continuum limit. A deconfining phase transition occurs as the cutoff is raised, even in the strong coupling limit. Abelian-projected monopoles in the maximal abelian gauge are strongly suppressed by the action restriction. Analysis of the steeply declining monopole loop distribution function indicates that for cutoffs $c > 0.5$, large monopole loops which are any finite fraction of the lattice size do not exist in the infinite lattice limit. This would seem to imply the theory lacks confinement, which is consistent with a fixed point behavior seen in the normalized fourth cumulant of the Polyakov loop.

11.15.Ha, 11.30.Qc, 5.70.Fh

I. INTRODUCTION

In some ways, placing a continuum theory on a lattice is a dangerous thing. The discreteness of space-time on the lattice results in fields which are discontinuous. If these discontinuities are small compared to the typical field magnitude, then a reasonable interpolation could be defined. However, often fields are so discontinuous as to be nearly random, and qualitatively different interpolating fields could be fit to them. Such large discontinuities can result in spurious effects that can exist only in the lattice theory and not in the corresponding continuum theory. For instance, large amounts of electromagnetic flux can be "lost" between the links of the lattice, creating large violations of Gauss' law. This combined with the compact nature of the gauge group can result in pointlike or stringlike topological defects on the smallest 1x1 scale: vortices, monopoles, and strings of monopole current. These defects can change the qualitative behavior of the theory, for instance in the U(1) theory monopoles disorder the theory so much that a confinement-deconfinement phase transition occurs at a coupling around $g = 1$, resulting in a dramatically different new phase on the lattice, a confining one, not connected to or relevant to the continuum theory.

The SU(2) theory on a finite lattice also appears to undergo a confinement-deconfinement phase transition, but this is usually interpreted as a finite temperature phase transition, one that exists if one of the four lattice dimensions is kept finite, and the other three are allowed to become infinite. This transition is expected to disappear in the 4-d symmetric infinite lattice limit. This finite-temperature phase transition has been linked to the U(1) bulk phase transition in the following way. If the SU(2) gauge configurations are transformed into the maximum abelian gauge and abelian projected to U(1) fields, then the SU(2) finite temperature deconfining transition is coincident with the monopole-induced transition for the corresponding U(1) fields [1,2]. In the confining phase there are large loops that tend to span the lattice, perhaps in a percolating cluster. When large loops are not present, the Polyakov loop shows deconfinement [3]. In addition, the monopole part of the U(1) field seems to carry most if not all of the SU(2) string tension [2]. The same is also true for the U(1) theory itself [4].

Although the monopoles are definitely artefacts in the U(1) lattice gauge theory, they are not necessarily artefacts in the SU(2) theory, where it is possible that topological objects larger than the 1x1 scale exist (fat monopoles and vortices) that survive the continuum limit, but which map into 1x1 scale monopoles and their associated thin Dirac strings when the abelian projection is performed [5]. However, there are also undoubtedly 1x1 scale objects in the SU(2) theory which *are* artefacts, and which also show up in the abelian projection. Thus it would seem important to find a way to eliminate or at least suppress these SU(2) artefacts, so the effects of the large topological objects relevant to the continuum limit can be seen. The remaining theory, with artefacts eliminated, could be substantially different. With less "noise" at the smallest scale, it may be possible to identify the so far elusive key properties of gauge configurations which are responsible for confinement.

II. RESTRICTED ACTION

The aim of this research is to study the effects of suppressing artefacts through ever greater restrictions on the action, beyond the positive-plaquette restriction. In SU(2) and

U(1) lattice gauge theory the usual Wilson action can be written as $1 - P$ where P is the plaquette variable which ranges from -1 to 1. Thus restricting the action to be less than a certain value is equivalent to restricting the plaquette to be greater than some cutoff value. The action to be used is the usual Wilson action with the added constraint that $P \geq c$, where c is some cutoff value. Since the continuum limit is determined only by the behavior of the action in an infinitesimal region around its minimum, which occurs around $P = 1$, this action should have the same continuum limit as the Wilson action for all cutoffs $c < 1$. The case $c = 0$ has been studied before as the positive plaquette action. The positive plaquette action eliminates a class of SU(2) artefacts, Z2 monopoles and vortices, which were once thought to possibly be the cause of confinement. Mack and Pietarinen found a much smaller and non-scaling string tension than in the standard action [6]. However the theory still confined, as confirmed later by Fingberg et. al. [7]. A similar action, the logarithmic action was introduced in [8]. It was also shown to confine at very strong couplings [9].

The efficacy of action restrictions in suppressing artefacts is demonstrated by the U(1) theory, for which a restriction $c > 0.5$ eliminates all monopoles. This can be seen as follows. In a monopole, 2π units of flux enter an elementary cube from a Dirac string. The flux splits up and emanates in all directions. Since the entering Dirac string is not "visible", this cube looks like a point charge. The flux coming out of the six plaquettes bounding the monopole must add up to the original 2π. Therefore, if the flux through each plaquette is forced to be less than $\pi/3$, through a plaquette constraint, such a monopole can not exist. Since $\cos(\pi/3) = 0.5$ this corresponds to $c > 0.5$. Monopoles have also been eliminated from U(1) by preventing the formation of strings [10]. The action restriction idea has also been applied successfully to spin theories to eliminate the effects of large discontinuities [11].

The above success in eliminating artefacts suggests trying restrictions of order $c = 0.5$ in the SU(2) theory as well. This would likely eliminate or at least suppress similar non-abelian objects, while allowing larger objects of any kind, such as fat monopoles, to still exist. For example, even with a restriction $c = 0.5$, the 2x2 Wilson loop can still take on any value. From this point of view such a restriction is not very severe. One is simply requiring each plaquette to carry less than 1/3 its maximum flux, which is a larger amount than could be carried in an unrestricted plaquette on a lattice with half the lattice spacing.

III. DECONFINING PHASE TRANSITION

To give the restricted action lattices a maximum chance to confine, Monte Carlo simulations were performed in the strong coupling limit $\beta \to 0$, i.e. the configurations are unweighted except that they obey the action restriction constraint. At least 1000 equilibration sweeps were performed, followed by from 10,000 (for the 20^4 lattices) to 1,500,000 (for some 6^4 lattices) measurement sweeps. Simulations were performed on a large number of Pentium PC's.

As the cutoff is raised, a deconfining phase transition occurs on all lattices studied. There is a fairly strong finite lattice size dependence, with the apparent "critical cutoff" at around 0.16 for the 6^4 lattice, 0.30 for the 12^4 and 0.37 for the 20^4 (these each have an uncertainty of about 0.01). It is therefore very important to determine the nature of the infinite lattice-size limit. If the transition is akin to a finite temperature transition, it will be forced to $c = 1$ as the lattice size goes to infinity. On the other hand if it is a 4-d percolation transition similar

to U(1) it will approach a limiting value of c which is less than unity, perhaps around 0.5. Percolation transitions can have substantial finite size dependences [12], so this could easily be confused with a finite temperature transition.

The behavior of the Polyakov loop shows the normal symmetry breaking behavior and is smooth, suggesting a continuous transition (Fig. 1). The lower curves in Fig. 1 are the modulus of the spatially averaged Polyakov loop, $< |L| >$, with the first (absolute value) moment of a Gaussian of the *same width* subtracted from it. This gives a sharper picture of the phase transition by correcting for the use of the absolute value of the Polyakov loop as the order parameter. In the confining phase, where the Polyakov loop distribution is very close to Gaussian, this subtracted Polyakov loop is zero within errors, whereas it is nonzero in the symmetry breaking region. Histograms show typical symmetry-breaking behavior of a higher order transition (Fig. 2).

The normalized fourth cumulant of the Polyakov loop, $g_4 \equiv 3 - <L^4>/<L^2>^2$, shows fixed point behavior (no discernible lattice size dependence) for $c > 0.5$ at a non-trivial value around $g_4 = 1.6$ (Fig 3). (The data for $c = 0.5$ on the 16^4 and 20^4 lattices are inconclusive as to whether dropping or not). This suggests either a line of critical points for $c > 0.5$, e.g. from a massless gluon phase, or that correlation lengths are so large that finite lattice size dependence is hidden. For the standard picture of all cutoffs being ultimately confining at zero temperature to hold, the normalized fourth cumulant should go to zero as lattice size approaches infinity for all values of $c < 1$. (Conversely, above a phase transition it should go to 2, and at a critical point some non-trivial value in between). The susceptibility also appears to diverge with lattice size for $c > 0.5$ [13], a further indication of criticality in this region.

Extrapolations of finite lattice "critical cutoffs" to infinite lattice size can also be attempted (Fig. 4). The critical cutoff can be defined many ways, and will have somewhat different values depending on the definition, because, after all, the finite lattice system is not really critical but just showing a rapid change in behavior. The method used for Fig. 4 was to extrapolate the subtracted Polyakov loop (defined above) to zero. This quantity is consistent with zero in the confining region, and rises above in the deconfined region. Quadratic fits, which fit the data well in the region just above criticality were used. The finite-lattice critical point, c^*, was defined as the point at which the extrapolation hits zero. Other definitions of finite lattice "critical cutoffs" give very similar looking graphs [13]. The finite lattice data can then be extrapolated to infinite lattice size ($N \to \infty$). A straightforward linear fit (excluding the 6^4 point) gives an infinite lattice critical cutoff of $c^*_\infty = 0.48$. A fit of the form $c^* = c^*_\infty + c_1 N^{-0.8}$ can fit all of the data and gives $c^*_\infty = 0.49$. However, it is possible that the graph will curve up sharply when extremely large lattices are encountered. As seen from the figure, a logarithmic scaling function of the form $c^* = 1 + a/\ln(b/N)$ can fit the data and has $c^*_\infty = 1.0$ in which case the transition would no longer exist on the infinite lattice. Therefore these data alone do not constitute a definitive test of the infinite lattice behavior. What is needed is a quantity that shows less finite lattice dependence, so that a more reliable extrapolation to the infinite lattice can be made.

IV. MONOPOLE LOOP DISTRIBUTION

The correlation of confinement with the appearance of large monopole loops suggests another more definitive approach to extrapolate to the infinite lattice. This concerns the loop size distribution function, which for this action appears to follow a simple power law, as it also appears to do for the Wilson action [14]. The power can be extracted from the behavior of small and mid-sized loops on finite lattices and appears to be independent of the lattice size. Once the power is known, then the probability of having loops of order the lattice size on lattices of arbitrary size can easily be predicted. Except for one limiting case, this probability will either vanish or diverge as the lattice size is taken to infinity, producing either a presumably deconfined or confined theory.

Gauge configurations from the restricted action simulations were transformed to maximum abelian gauge using the adjoint field method [15]. It was found that this worked optimally when the adjoint field was recalculated after each sweep of the gauge field. Abelian monopole currents were then extracted in the usual way using the DeGrand-Toussaint procedure [16]. Sample sizes ranged from 500 configurations for some 20^4 lattices, to $200,000$ for some 6^4 lattices. The imposition of the cutoff produces a rather severe suppression of monopoles, the density of which is shown in Fig. 5. The data are consistent with an exponential suppression of the form $\rho \propto \exp(-k/(1-c))$ as shown by the fits in the figure. The 12^4 fit gives $k = 15.8 \pm 0.3$. A moderate finite-size shift is seen for the 6^4 data, but there is not much difference between the 12^4 and 20^4 data. If this exponential continues for larger c, then some monopoles will exist for any value of c, making it possible for some to survive the continuum limit.

For $c > 0.5$ most lattices of practical size have no monopoles, e.g. at $c = 0.53$ only about one out of every 1000 12^4 lattices has *any* monopoles, usually a single minimal loop of size four. Of course, even with this low density, the infinite lattice will still have an infinite number of monopoles. However, what is important is whether they form into large loops, because only these configurations can disorder large Wilson loops to produce confinement. Small loops, such as the most common minimal loop of size four, will have zero physical size in the continuum limit and presumably no effect on physics. Whether large loops exist on large lattices depends on how fast the probability of finding loops of size l (i.e. length l) decreases with l. Define the loop distribution function, $p(l)$, as the probability (normalized per lattice site) of finding a monopole loop of size l on a lattice (of any size). Evidence will be presented that $p(l)$ is independent of lattice size, for l less than several times the linear lattice size, N. The probability of finding a loop of size N *or larger* on an N^4 lattice is given by $N^4 I(N)$ where $I(N)$ is the integrated loop distribution function

$$I(N) = \int_N^\infty p(l)dl \tag{1}$$

where, since N will be taken large, the discrete distribution has been replaced by a continuous one. To get confinement, at least some finite fraction of lattices would have to contain loops of size order N or larger. (Some would argue that loops of size N^2 or larger might be necessary to get a linear extent of order N, due to the crumpled nature of the loops). Conversely, if

$$\lim_{N\to\infty} N^4 I(N) = 0 \tag{2}$$

then there will be no loops of size N *or any finite fraction* of N present on the N^4 lattice in the large lattice limit, and the lattice will almost certainly be deconfined (assuming that large monopole loops are necessary for confinement).

In Fig. 6, $\log_{10} p(l)$ is plotted vs. $\log_{10}(l)$ for various cutoffs and lattice sizes. The data are consistent with a power law, $p(l) \propto l^{-q}$, for loops up to around size $l = 3N$ (the size 4 loops, which fall slightly below the trend are excluded from all fits). The larger the lattice the further the power law is valid before some deviation at large l. Also note that the 12^4 and 20^4 data are virtually identical for loops up to size 30 or so. Linear fits were made for loop sizes in the range 6 to 2N, or less if the data had run out (only the 12^4 fits are shown for clarity). For larger loop sizes, occasionally zero instances of a particular size was observed. These cannot be plotted, but if they are ignored the data will be biased upward. A running average procedure was used in this circumstance to properly account for the zero observations. For the most part this was beyond the region where fits were performed. For $c = 0.51$, the data were insufficient to give a reasonable two-parameter fit. Instead, for this case the constant term was predicted from the trend observed for the constant terms of the other fits, and a one-parameter fit was made for the slope.

The deviations from linearity for large loops can be easily understood as a finite size effect coupled with the periodic boundary condition. For loops longer than about $2N$, there is a significant probability of reconnection through the boundary. This makes a would-be large loop terminate earlier than it would on an infinite lattice. Thus, on a finite lattice there will be a deficit of very large loops, and an *excess* of mid-size loops due to this reconnection effect. Looking at the data e.g. for $c = 0.30$, it is apparent that this is indeed happening. The linear trend continues further for the 20^4 lattice than for the 12^4. The observed data does fall below the trendline for very large loops in the sense that zero instances of loops beyond those plotted occurred. Of course these cannot be plotted on the logarithmic graph, but the consequences can be taken into account in the following way. If one assumes that very large loops follow the trendline in the figure for the 12^4 data, then one can calculate that 5.3 instances of loops in the size range 342 to 2000 should have been seen in the sample. The fact than none occurred implies rather strongly that the data does eventually fall below the trend line for very large loops ($p < 0.01$). Similar arguments can be applied to the other data samples.

Because the power law trend continues further the larger the lattice, and the deviations are easily understood as a finite size effect, it seems quite reasonable to assume that on the infinite lattice one would have a pure power law. It is difficult to imagine what could set the scale for a significant change in behavior at extremely large loop sizes beyond those measured here. In addition, since the small loop data are nearly independent of lattice size, it would seem the power must also be essentially the same on the infinite lattice as observed here for the 12^4 or 20^4 lattices. Assuming this, one can easily predict the point at which condition (2) becomes satisfied, namely $q > 5$. For $q > 5$ the probability of having a monopole loop with length equal to any finite fraction of the lattice size N vanishes in the large lattice limit, whereas for $q < 5$ the same becomes overwhelmingly likely.

The power q is plotted as a function of cutoff in Fig. 7. A definite rising trend is observed, with q passing 5 around $c = 0.45$. It is very difficult to gather enough statistics for $c > 0.5$ since monopoles are extremely rare here, however if all of our runs in this region are combined(c=0.51 to 0.55), then the following statement can be made. Out of a total

sample of over 10 billion links, only a single loop of size 8 was found (at $c = 0.51$), and none larger, whereas 230 loops of size 4 and 36 of size 6 were found. If $q \leq 5$ then the expected number of size 8 loops, given this many size six loops, would be at least 8, and several even larger loops should have been seen. Using Poisson statistics, the probability of obtaining our result for size 8 and larger loops if $q \leq 5$ can be computed to be around 10^{-5}. Thus it appears overwhelmingly likely that q exceeds 5 for $c \geq 0.51$. Therefore, the loop distribution function strongly supports the notion that this theory is deconfined in the infinite lattice limit for $c > 0.5$. This is in concert with the results from the fourth cumulant of the Polyakov loop (Fig. 3), showing fixed-point behavior in this range, and with the straightforward extrapolation of critical cutoffs (Fig. 4).

The rather strong finite lattice size dependence of the critical cutoff in this theory, or critical β in the standard Wilson-action theory can be understood from the following argument. Say that the theory becomes confining at the point that $N^4 I(N) = 0.5$, i.e. when 50% of the lattices of size N have a loop of length at least N (the exact criterion is irrelevant). In the region $q < 5$, the LHS is an increasing function of N, so it will be satisfied for some N. If c is raised, then q will increase, requiring a larger N to stay on the transition. If q varies relatively slowly with c then there will be a substantial change in the critical value of c as N is changed, until q gets close to 5, at which point the critical cutoff will reach its limiting value. It is very likely that changing β in the standard theory has a similar effect to changing c. To test this some preliminary runs were performed with the standard Wilson action on a 12^4 lattice. Contrary to the suggestion in [14], a rather substantial dependence of q on β is found, with $q \approx 3$ at $\beta = 2.4$ and $q \approx 5$ at $\beta = 2.9$. These data predict that the standard Wilson-action theory will be deconfined on the infinite lattice for all $\beta > 2.9$. Details will appear in a separate report, when greater statistics become available.

V. DISCUSSION

The above results suggest that the possibility that the continuum SU(2) pure gauge theory may not be a confining theory needs to be taken seriously. This has been suggested before [17–19]. In fact Ref. [19] predicts that a sufficiently strong plaquette restriction would result in a theory that is non-confining for all couplings and temperatures.

Of course, one could instead give up the link between abelian monopoles and confinement, despite the overwhelming evidence in favor of a connection. Since abelian monopoles have been shown to be responsible for most if not all of the SU(2) string tension, if the theory still confines when they are removed it will be a more subtle form of confinement, with a likely much smaller string tension. However string tension is probably not the best test of whether a lattice is confining or not. The Polyakov loop is a much better order parameter for confinement simply because it does concern an actual symmetry breaking. It is very difficult to tell if a string tension is exactly zero due to other terms in the potential and the functional forms assumed for them [20]. There is no problem defining the Polyakov loop on a symmetric lattice and, although it does go to zero in both phases as the lattice size $N \to \infty$, one can still look for symmetry breaking at any large finite N, as large as one likes, or take the N^{th} root and then the limit $N \to \infty$. Normalized cumulants such as g_4 will also have nontrivial $N \to \infty$ limits that allow one to distinguish broken from unbroken symmetry behavior by looking for non-Gaussian behavior in the limit of large

lattices. The case for deconfinement in the restricted action theory from the Polyakov loop and its moments alone is fairly compelling, though not as definite as the monopole loop data, due to the large finite size dependence of the critical cutoff. Nevertheless, it supports the notion that deconfinement results when large abelian monopole loops disappear.

Our simulations are in the strong coupling limit. Letting β grow larger than zero will further order the theory. If the theory is already deconfined in the strong coupling limit, it is very unlikely that confinement could come back as the coupling is weakened. Thus the zero-temperature continuum limit, $\beta \to \infty$ would also be deconfined.

The behavior of the standard theory in the fundamental-adjoint plane can also be interpreted as supporting this conjecture, as it suggests that what is normally thought of as a finite-temperature transition may actually be a zero-temperature bulk transition, since it appears to connect to a previously known bulk transition [21]. Although some evidence of a separation of the bulk and finite temperature transitions has been presented [22], this can be interpreted merely as a manifestation of the fact that different methods of finding a critical point on a finite lattice will usually give slightly different values, agreeing only in the thermodynamic limit. There has yet to be a simulation showing two distinct transitions at different β's on the same lattice. By bulk transition, it is meant here a transition that remains at finite β in the infinite (4-d) lattice limit (as opposed to a finite-temperature transition for which $\beta_c \to \infty$ in this limit). If the deconfinement transition is a percolation transition similar to U(1) this will be true. However percolation transitions differ in one respect from what is usually called a bulk transition in that only a fractal network of links comprising a small fraction of the total set of lattice links actually participates in the transition, so scaling properties will likely differ from a conventional bulk transition in which all plaquettes participate. This may explain the different from bulk scaling exponents seen in [23] for the first-order transition seen in the case of a large adjoint action. It could also explain the larger than normal finite-size shift in critical point for the standard Wilson action theory, since this shift is related to the scaling exponents.

It is important to ask the question of whether or how continuum QCD could live with a non-confining continuum SU(2) theory. First, the behavior of SU(3) could differ. Although this certainly needs to be checked, there has always been a qualitative agreement between these theories so far. Another possibility is that confinement is not absolute, in the sense of a linearly rising potential that goes on forever. All that is needed in the real world is for the potential to have a nearly linear portion in the range 1-5 fm. Beyond this, particle pair creation causes the "string" to break in the real world, so details of the potential at larger distances in the pure gauge theory are irrelevant to experiment. A logarithmic running coupling can modify the Coulomb potential to produce a potential that is nearly linear in this range, but at large distances goes to a constant [20]. This may be enough to fit heavy quark spectroscopy.

Another possibility is that confinement could be due to chiral symmetry breaking [17,24]. With light fermions present it is likely that chiral symmetry will still break in a non-confining theory, since the coupling is strong. Confinement could then result from a polarization of the chiral vacuum which results in a higher than normal vacuum energy density in the region surrounding a colored object, including color dipoles such as mesons and baryons. This region of polarized vacuum moves around with the meson or baryon adding to its dynamical mass. Any attempt to stretch the hadron will stretch this "disturbed vacuum"

bag leading to an energy proportional to the elongation, i.e. a linear potential. This picture is consistent with the observation that $<\bar{\psi}\psi>$ is lowered in the neighborhood of a color source [25], indicating some expulsion of condensate. Since the condensate is expelled, the energy density must be increased, supporting the above picture, in which chiral symmetry breaking, confinement, and dynamical mass generation of quarks are all due to the same mechanism. This scenario is similar to the picture that emerges in chiral quark models [26] where a polarized Dirac sea is responsible for the binding of the quarks in a baryon, and also in the instanton liquid model [27]. Both of these models are able to compute with fair accuracy a large number of low-energy properties of hadrons, and neither has an absolutely confining potential.

As a final note, from the point of view of practical simulations, it may be better to take an action that is cut off more smoothly than the one considered here. A smooth cutoff action that disallows plaquettes smaller than a cutoff c is

$$S_{\Box} = \begin{cases} -(1-c)\ln\left[(P-c)/(1-c)\right] & \text{if } P > c \\ \infty & \text{if } P \leq c \end{cases} \tag{3}$$

where P is the plaquette. The smoothly cutoff action may have better scaling behavior, as has been shown for a similar logarithmic action based on the positive plaquette action [9].

VI. CONCLUSION

The imposition of a plaquette restriction causes a deconfining phase transition in SU(2) lattice gauge theory, even in the strong coupling limit. The critical cutoff is dependent on lattice size. Straightforward extrapolation as well as the behavior of the fourth moment of the Polyakov loop suggest that the infinite lattice critical cutoff will be around $c = 0.5$, the same value for which the U(1) theory must deconfine. This means that the theory will be deconfined on all symmetric lattices for $c > 0.5$, for any β. The abelian monopole loop distribution function confirms this by showing a power-law falloff with loop length l faster than l^{-5} for $c > 0.5$, from which it can be shown that no loops large enough to cause confinement exist on any size symmetric lattice. Light dynamical quarks may be a necessary ingredient to obtain a continuum confining theory.

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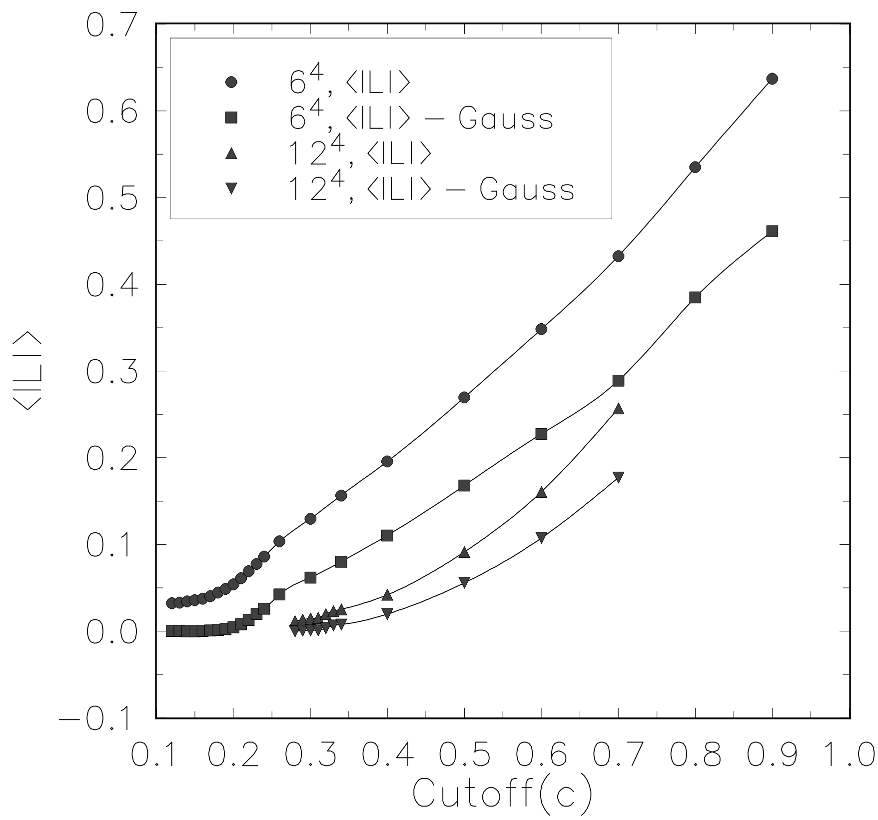
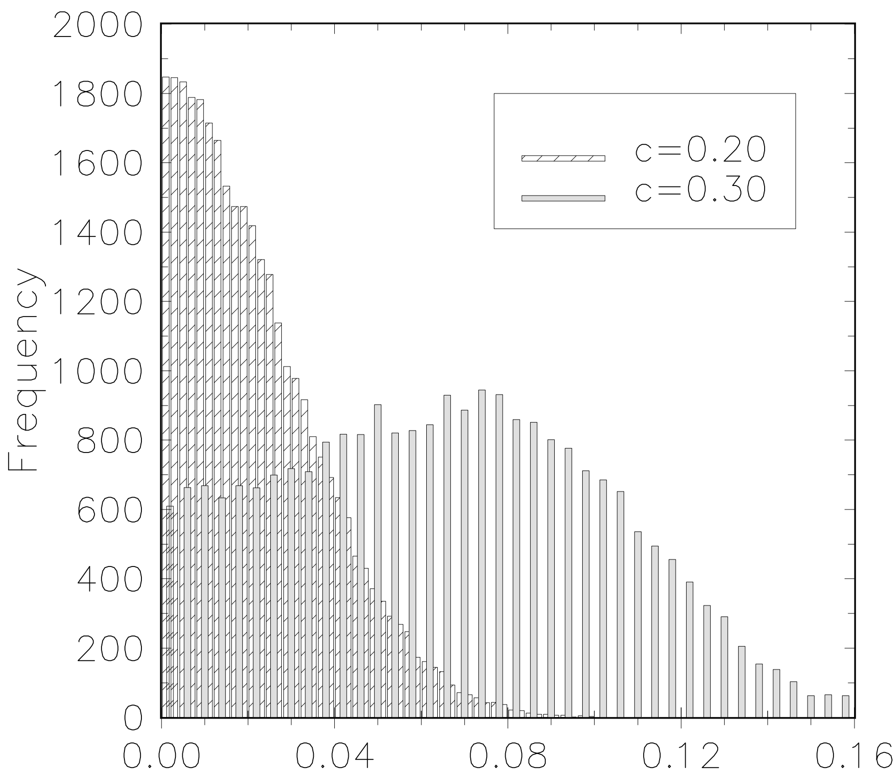
Figure Captions

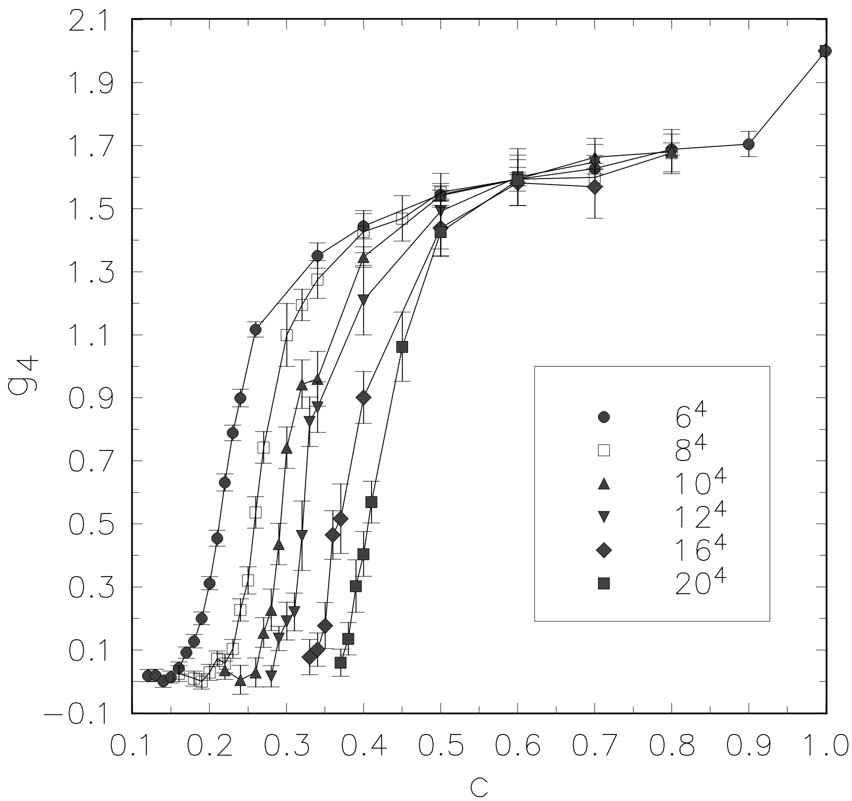
FIG. 1. Typical modulus of the Polyakov loop.
FIG. 2. Polyakov loop histograms for confined and deconfined 8^4 lattices.
FIG. 3. Normalized Fourth cumulant of the Polyakov loop. Errors are from binned fluctuations.
FIG. 4. Extrapolation of critical point to infinite volume. Uncertainties are about the size of plotted points. N is the linear lattice size. The short-dashed line is a linear fit, longer dashed line a fractional power fit, and the solid line is a fit to a logarithmic function given in the text.
FIG. 5. Logarithm of the monopole (plus antimonopole) density (number per lattice link) vs. $1/(1-c)$. A linear fit to the 12^4 data is also shown.
FIG. 6. Log-log plots of loop probability (per lattice site) vs. loop length. The six data series shown are, from right to left, $c = 0.30$, 0.36, 0.42, 0.45, 0.49, and 0.51. Trend lines explained in text are given for the 12^4 data. Only 12^4 runs were performed at the highest two cutoffs. Size four loops fall below the trend and are not included in fits.
FIG. 7. Power, q, describing the falloff of loop probability with loop length, vs. the cutoff, c. Line is drawn through the 12^4 data to guide the eye.



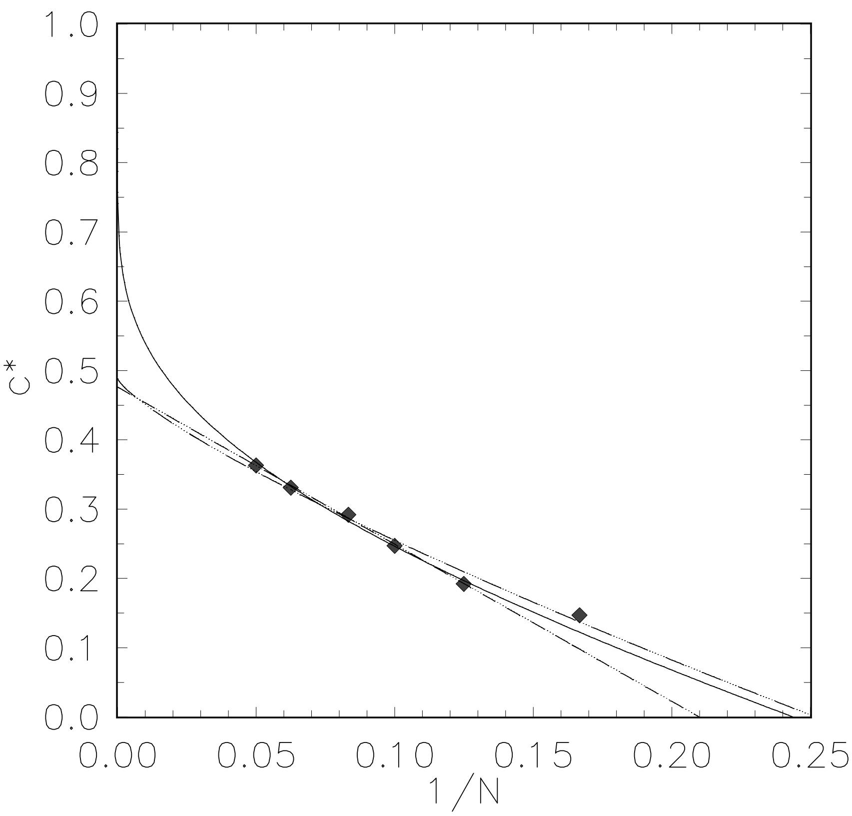
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
Cutoff(c)
⟨|L|⟩
6^4, ⟨|L|⟩
6^4, ⟨|L|⟩ − Gauss
12^4, ⟨|L|⟩
12^4, ⟨|L|⟩ − Gauss



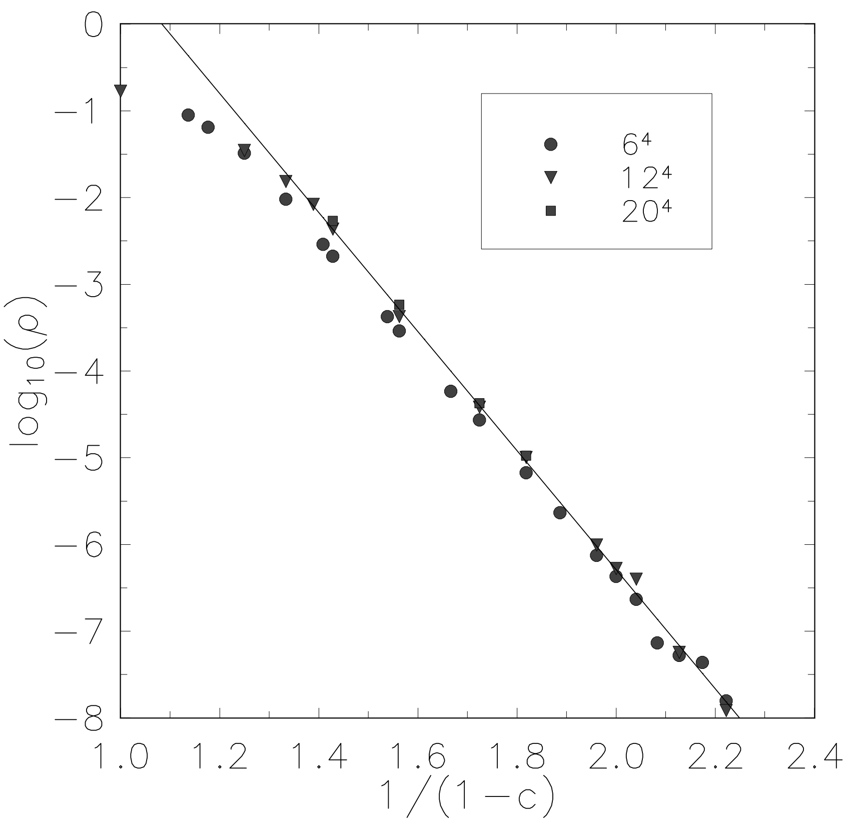
Frequency
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0.00
0.04
0.08
0.12
0.16
⟨|L|⟩
c=0.20
c=0.30



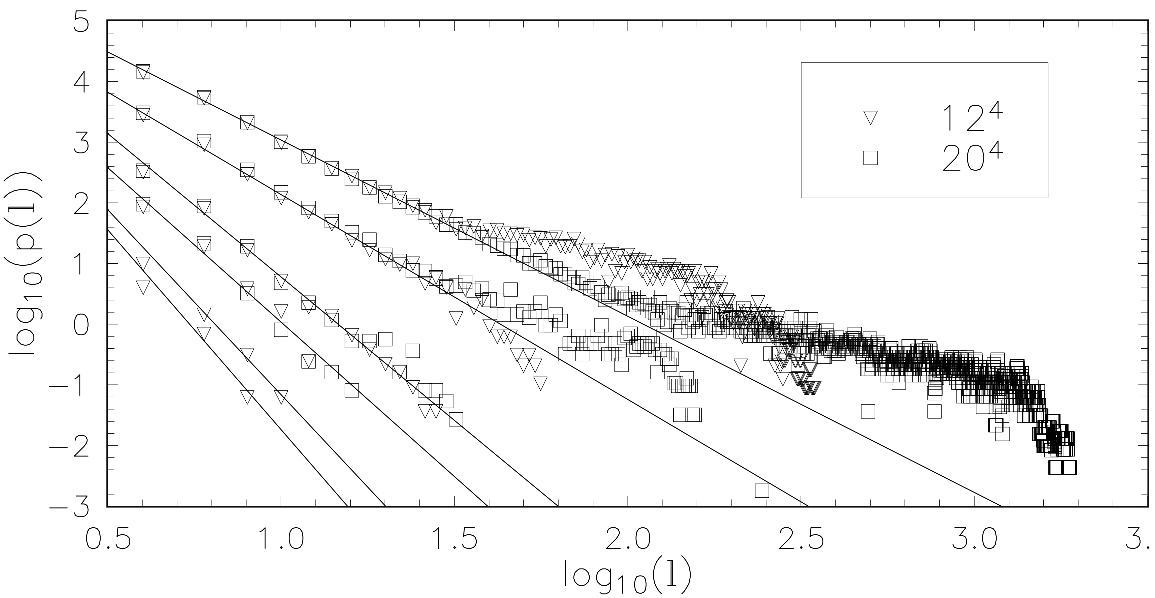
2.1
1.9
1.7
1.5
1.3
1.1
0.9
0.7
0.5
0.3
0.1
−0.1
g4
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
c
6^4
8^4
10^4
12^4
16^4
20^4



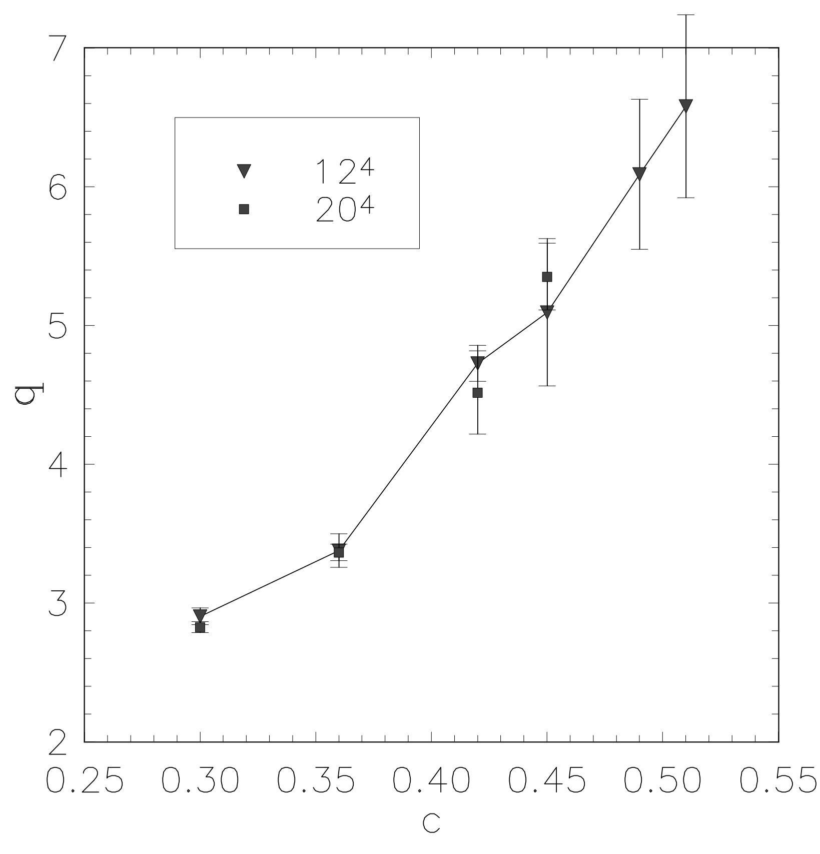
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
c*
0.00
0.05
0.10
0.15
0.20
0.25
1/N



0
−1
−2
−3
−4
−5
−6
−7
−8
log10(ρ)
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
1/(1−c)
6⁴
12⁴
20⁴



5
4
3
2
1
0
−1
−2
−3
0.5
1.0
1.5
2.0
2.5
3.0
3.5
log10(p(l))
log10(l)
12^4
20^4



7
6
5
4
3
2
q
0.25
0.30
0.35
0.40
0.45
0.50
0.55
c
12^4
20^4